



Why does Fractions Knowledge Support Algebra Knowledge? Investigating Multiple Paths



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Background and Research Question

Fractions skills predict students' success in algebra, but why? (e.g., Siegler et al., 2012)

- Fraction *magnitude* and *arithmetic* scores predict students' algebra scores. (Barbieri et al., 2021; Booth et al., 2014)
- *Units coordination* relates to students' algebraic reasoning. (e.g., Hackenberg et al., 2013)

1. How do these types of fractions knowledge relate?
2. Which of these types of fractions knowledge is a stronger predictor of 8th graders' algebra knowledge?

Method

US 8th graders
(N = 49)

Age = 14 yrs
23 female, 24 male
76% White

3 sessions on Zoom

Session 1 (Covariates)

- Working Memory (WM), Raven's Matrices
- Nonsymbolic Ratio Comparison
- Whole Number Estimation & Fluency

Session 2 (Fractions)

- Number Line Estimation & Comparison
- Fraction Arithmetic
- Schemes & Units Coordination

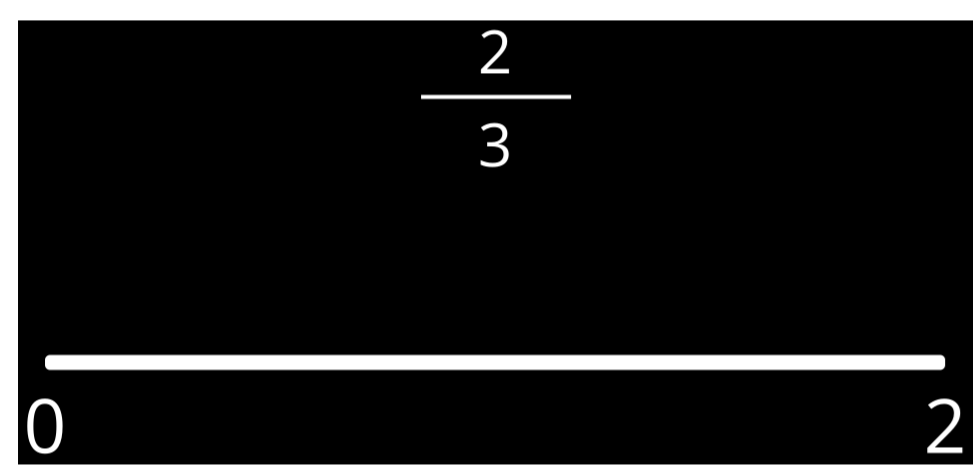
Session 3 (Algebra)



Fractions Measures

$$\frac{8}{17} \quad \frac{2}{15}$$

Comparison



Number Line Estimation (NLE; 0-1, 0-2, 0-5)

$$\frac{3}{5} + \left(\frac{3}{10} \times \frac{4}{15} \right) =$$

Arithmetic

Units Coordination (UC)



Pretend the **purple bar** fits into **orange bar** exactly 2 times.

Pretend the **green bar** fits into **purple bar** exactly 6 times.

How many times does the **green bar** fit into the **orange bar**?

This stick is 2/3 of a whole stick.
How many 1/9 sticks can you make from the 2/3 stick?

Algebra Measures

Which example could represent a linear function?

x	-3	0	3
y	4	6	8

$\frac{5}{x} + y = -7$

x	1	3	5	3
y	4	2	0	-2

$x + \frac{2}{y} = 4$

Conceptual Knowledge

Solve the equation for y. Show your work on paper and type your answer here.

$$5(y - 2) = -3(y - 2) + 4$$

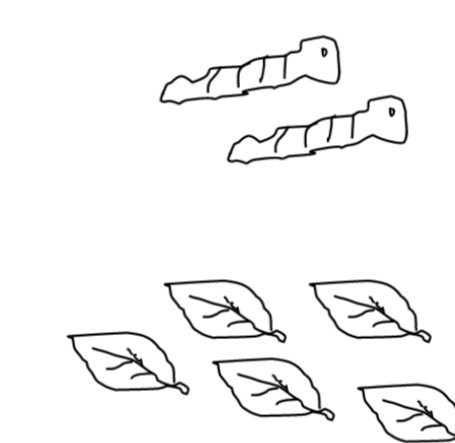
Procedural Knowledge

Below is the beginning of Gabriella's, Jamal's, and Nadia's work in solving $x + 7 - 3 = 12 - 2x$.

Gabriella's way: Subtract 3 from 7: $x + 4 = 12 - 2x$	Jamal's way: Add 2x to both sides: $3x + 7 - 3 = 12$	Nadia's way: Subtract (7 - 3) from both sides: $x = 8 - 2x$
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To start solving this problem, which way(s) may be used?

Flexibility

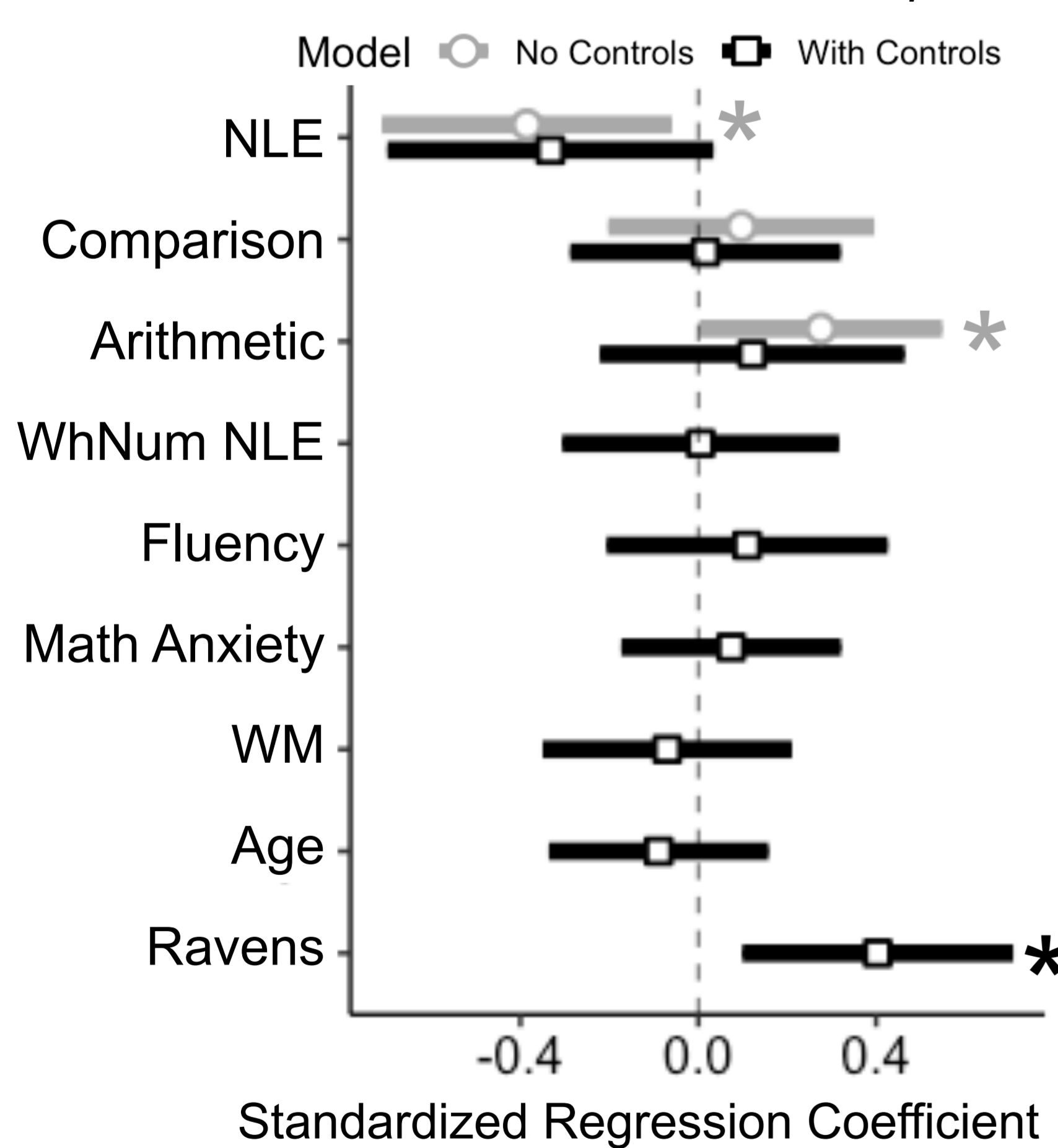


A class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would they need each day for 12 caterpillars?

Proportional Reasoning

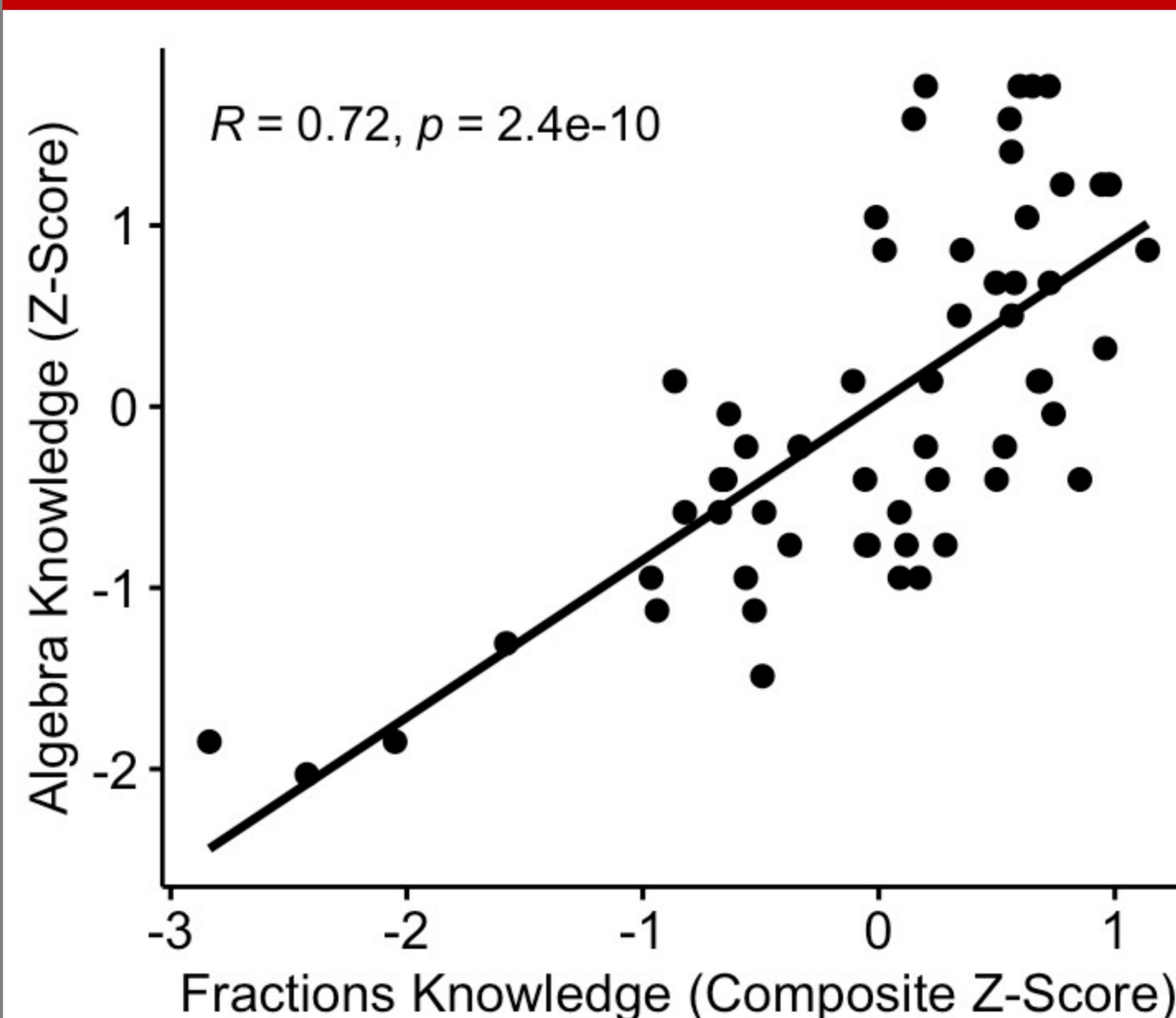
Results: Predicting UC

* $p < .05$

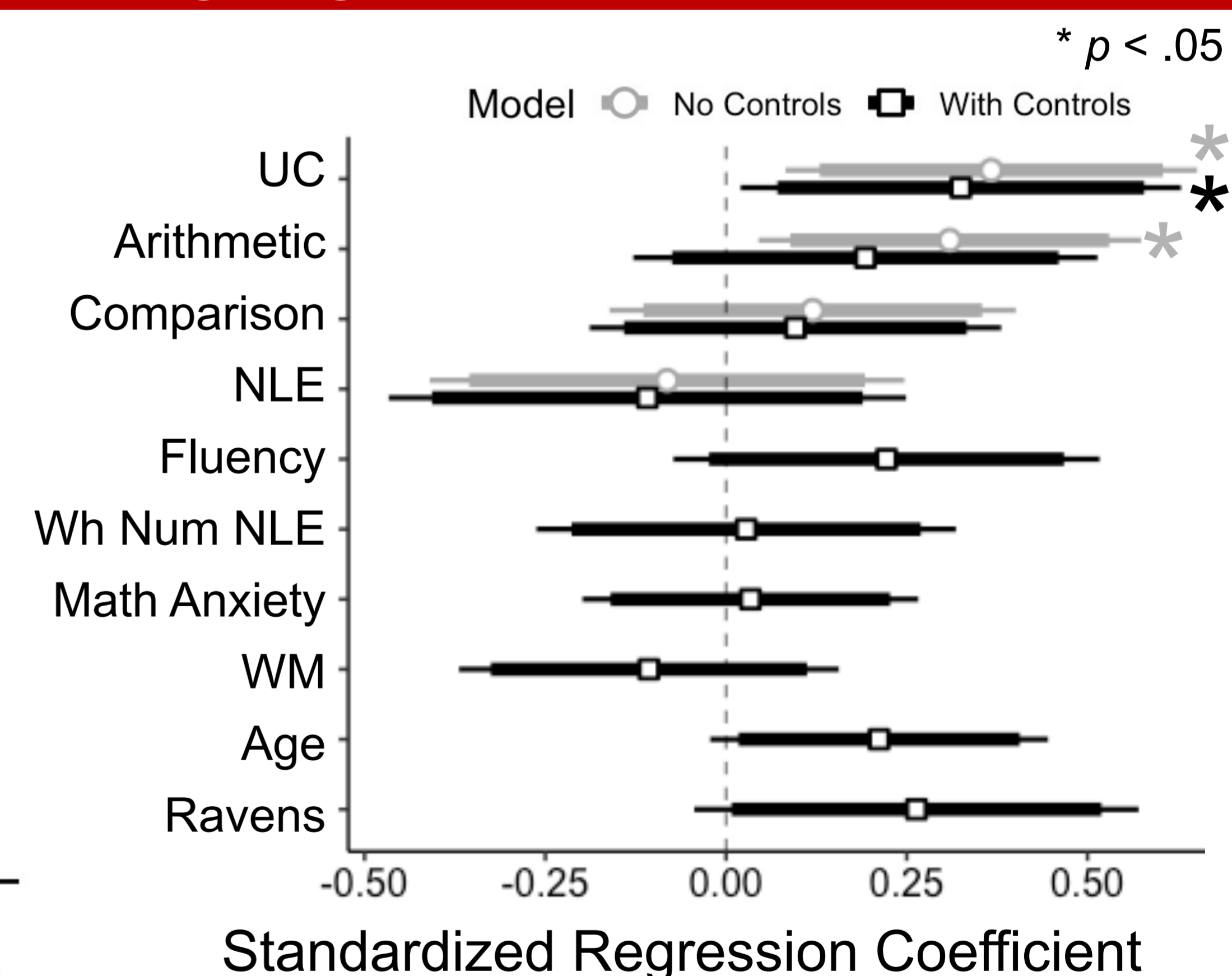


NLE and Arithmetic were related to UC, but only without controls.

Results: Predicting Algebra



Overall fractions knowledge predicted overall algebra scores ($\beta_{std} = .55, p < .001$), when controlling for other cognitive skills.



Units Coordination, but not other aspects of fractions, uniquely predicted overall algebra knowledge ($p = .03$).

Discussion

- For the first time, we show that children's **units coordination** predicts their algebra performance **even controlling for** other fractions skills, whole number knowledge, and domain-general cognitive skills.
- Future work should (1) examine children's **explanations** on these tasks and (2) **test mechanisms** experimentally.