How does fractions knowledge support algebra knowledge?

An interdisciplinary investigation

AERA 2023

Alexandria A. Viegut, Ana C. Stephens, Percival G. Matthews

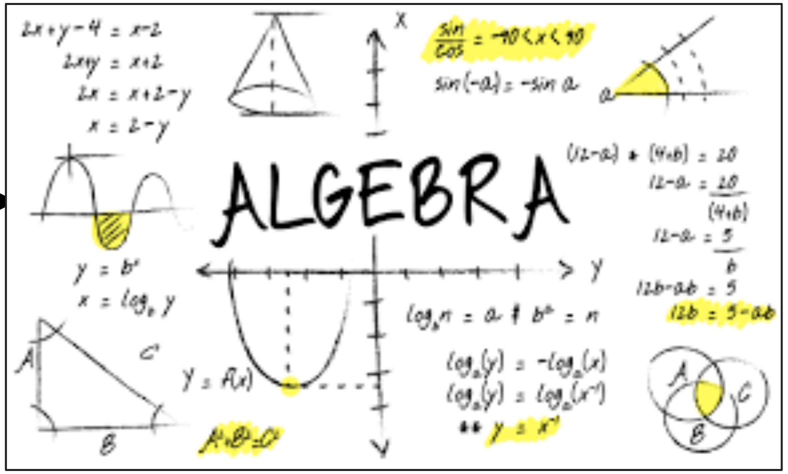
Fractions & algebra knowledge are closely related

- Correlated in many age groups (Hurst & Cordes, 2018; Powell et al., 2019)
- Fraction scores predict algebra scores 5-6 years later, controlling for other math skills, reading, demographics, etc. (Siegler et al., 2012)
- Fraction scores predict how much students learn from algebra instruction (Booth et al., 2014)

Fractions may be a key to the gate

Fractions Knowledge

Why?



Quantitative studies: Magnitude + Arithmetic

Fractions Knowledge → Algebra Knowledge

Higher Scores

Magnitude

Where is 3/12?

0 ——————— 1

$\frac{7}{8}$ $\frac{3}{5}$

Arithmetic

$\frac{2}{3} + \frac{5}{6}$ $\frac{2}{5} + \frac{3}{4}$ $\frac{1}{2} + \frac{4}{7}$

Higher Scores

$3(2x - 1) + 2x = 21$

What is the value of x?

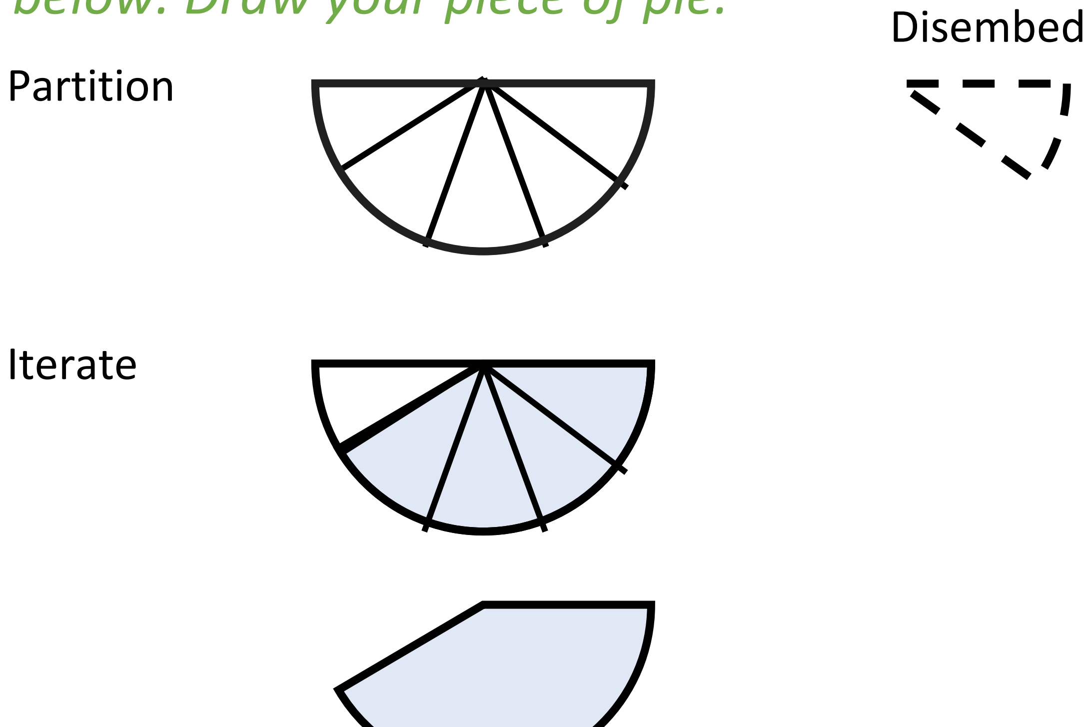
a	*b*
3	7
2	5
0	1

What is the rule?

Booth et al., 2014; Booth & Newton, 2012; Hurst & Cordes, 2018a; etc.

Qualitative studies: Fraction schemes

Your piece of pie is 4/5 as big as the piece shown below. Draw your piece of pie.



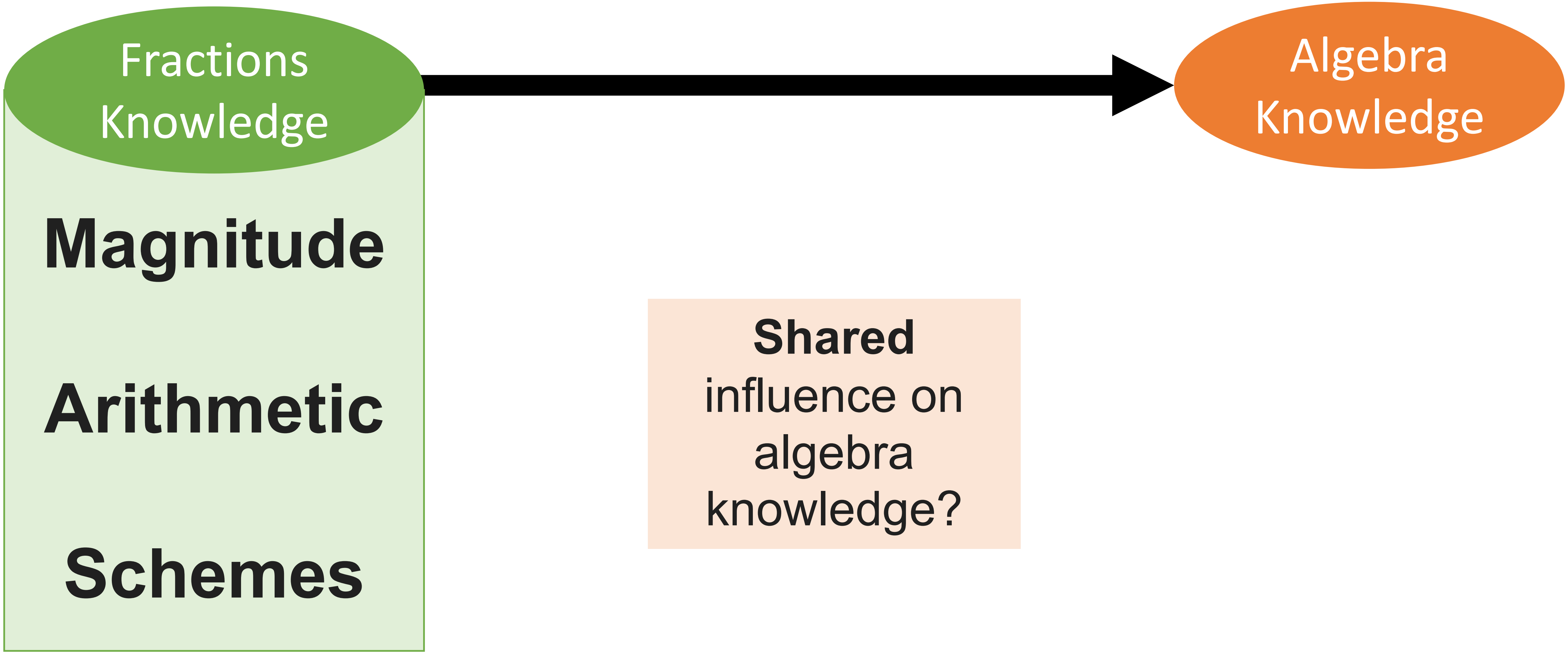
Wilkins et al. (2013)

Stephen's cord is five times as long as Rebecca's cord.
Can you write an equation for this situation?

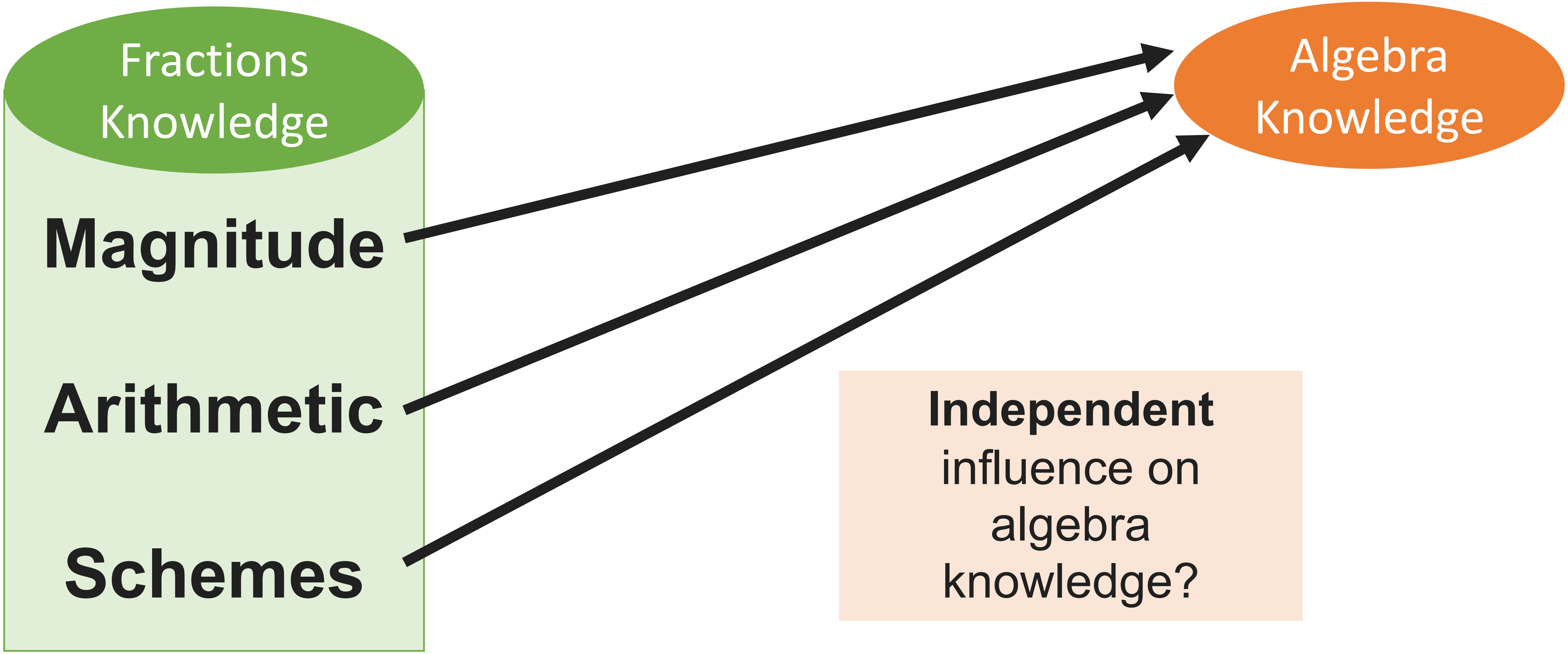
$$S = 5 \times R$$
$$R = S \div 5$$

e.g., Hackenberg & Lee (2015)

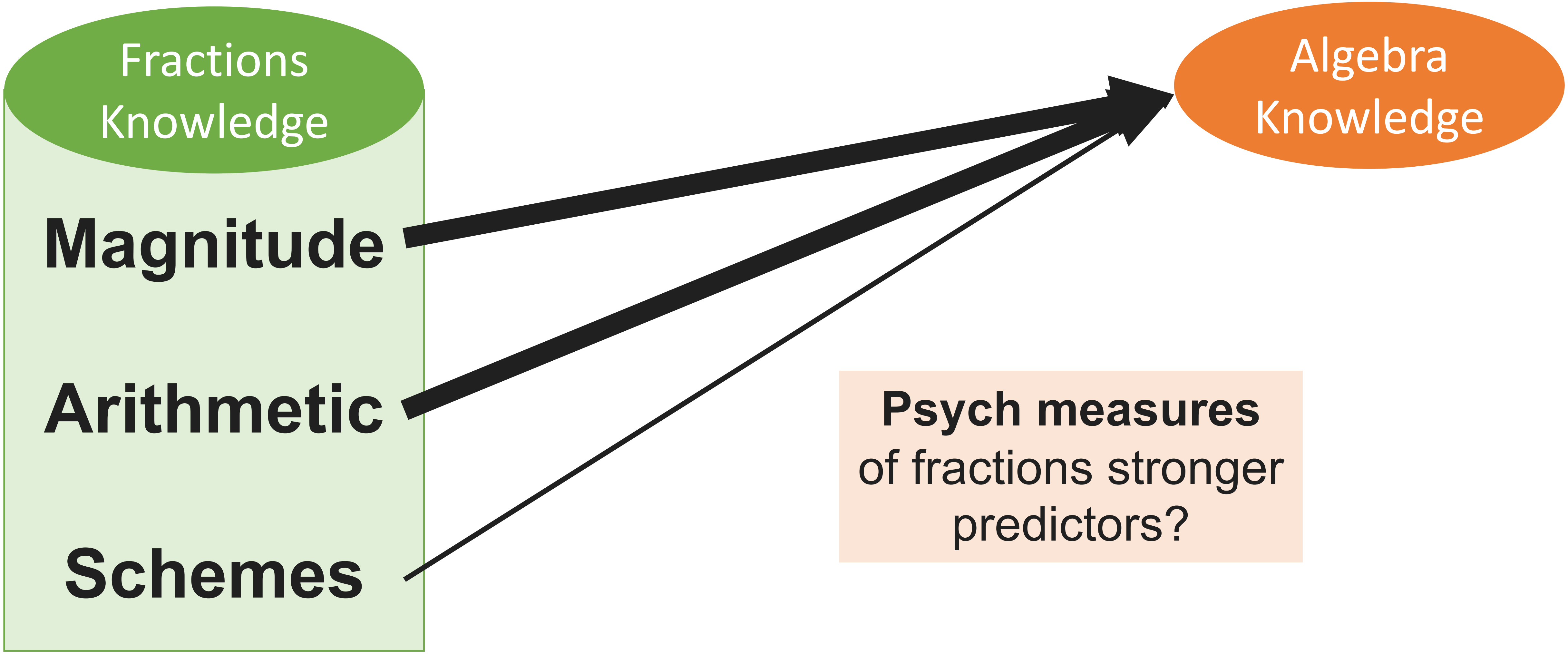
Which aspect(s) of fractions are most important?



Which aspect(s) of fractions are most important?

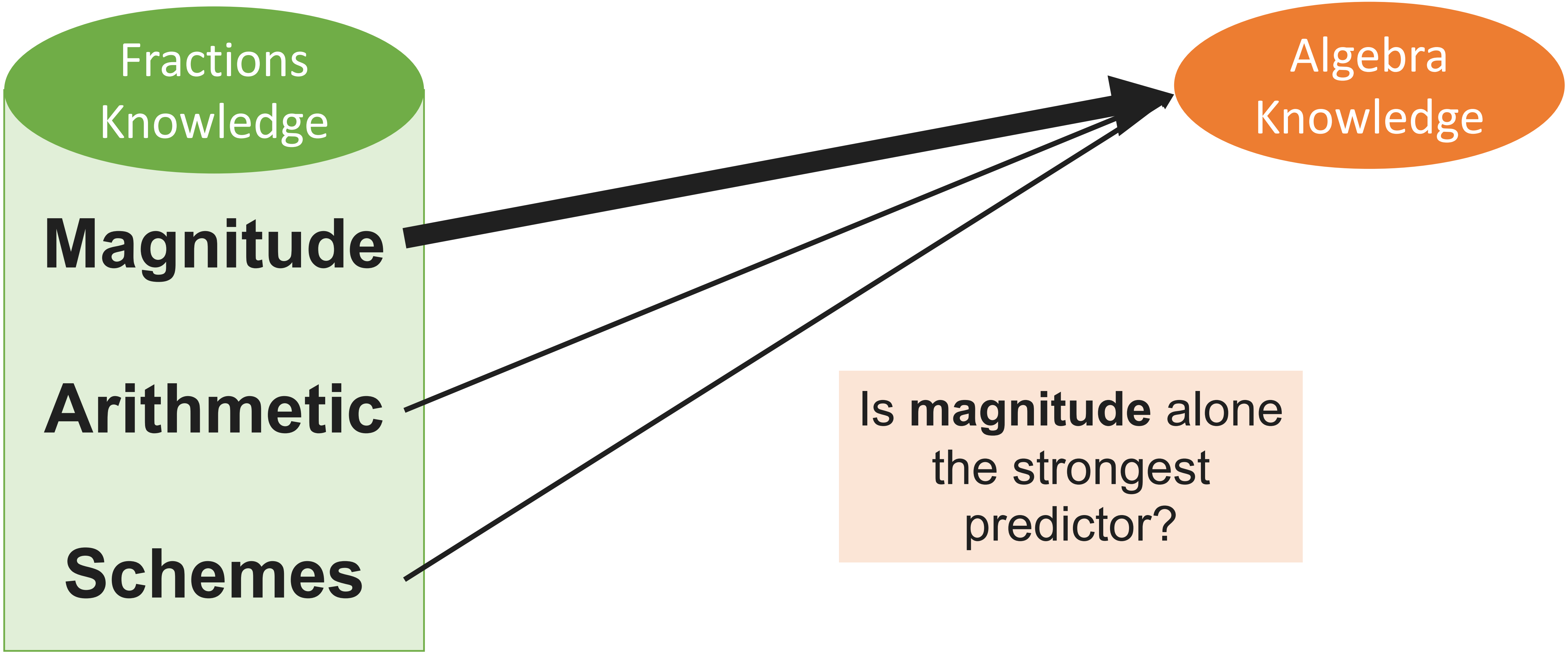


Which aspect(s) of fractions are most important?



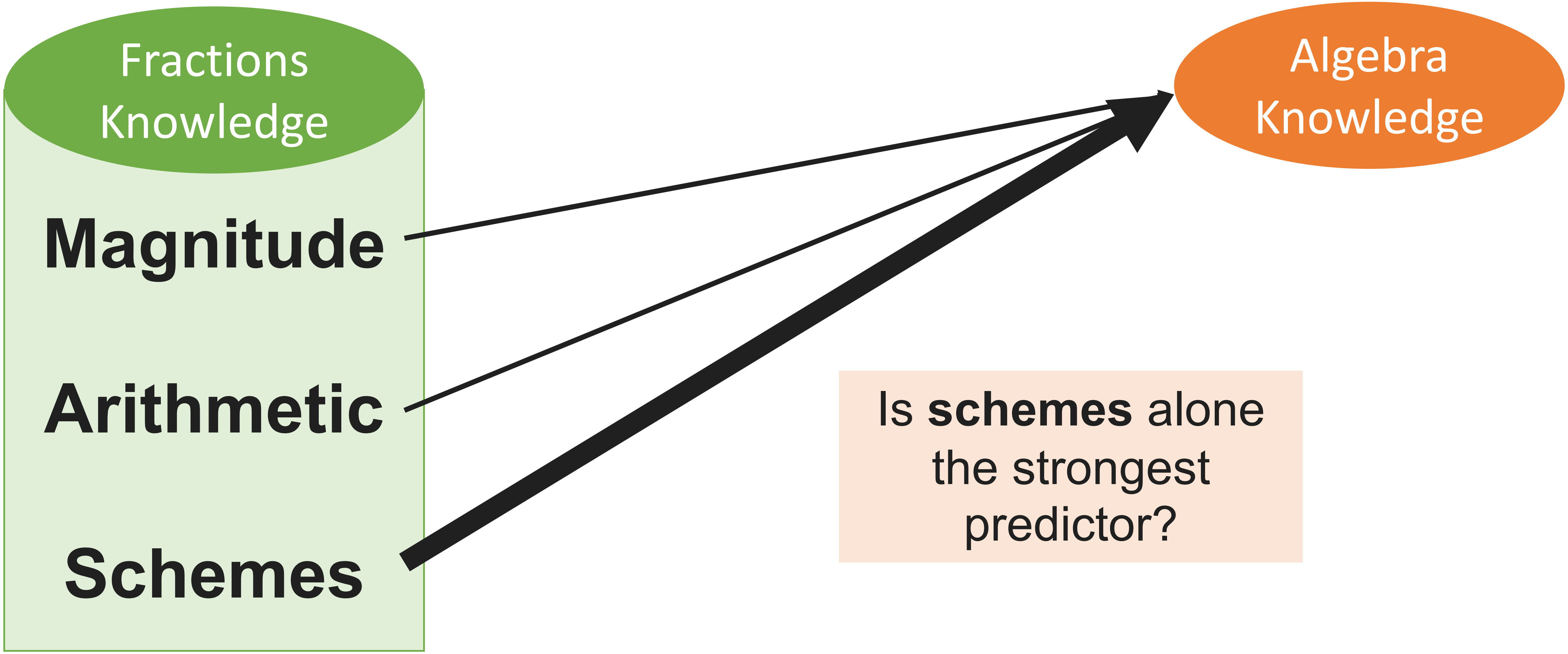
Psych measures of fractions stronger predictors?

Which aspect(s) of fractions are most important?



Is **magnitude** alone the strongest predictor?

Which aspect(s) of fractions are most important?

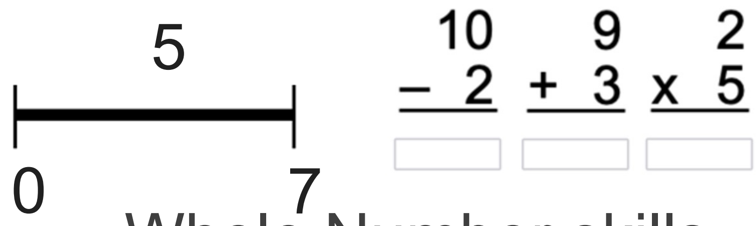


Is **schemes** alone the strongest predictor?

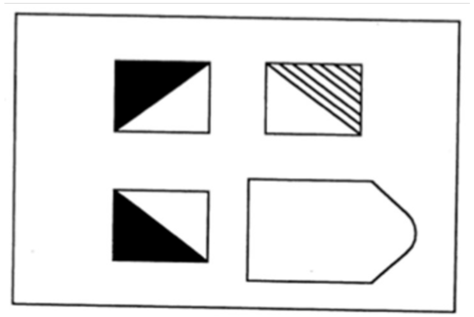
Study Design

8th grade students (N = 59) participated in 3 Zoom sessions.

Covariates



Whole Number skills



General cognitive skills

Math anxiety

Units Coordination

Fractions

-Magnitude

-Arithmetic

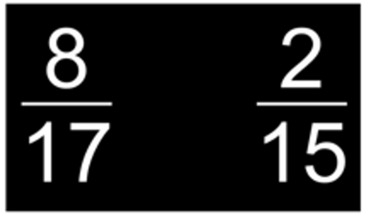
-Schemes

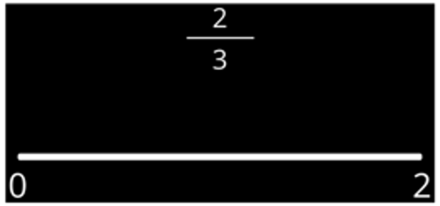
Algebra

45-minute test, Mix of multiple choice & open-ended

Fractions

Magnitude



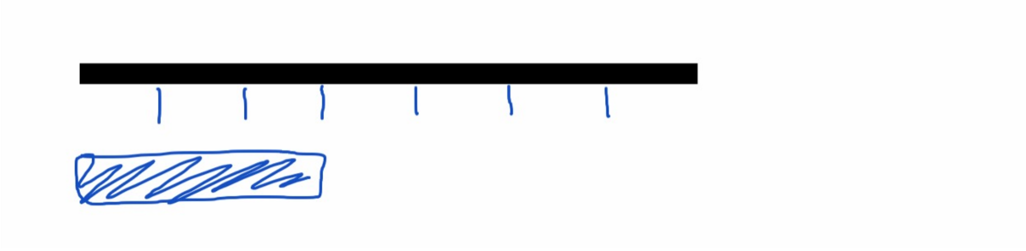


Arithmetic

$$\frac{3}{5}+\left(\frac{3}{10}\times\frac{4}{15}\right)=$$

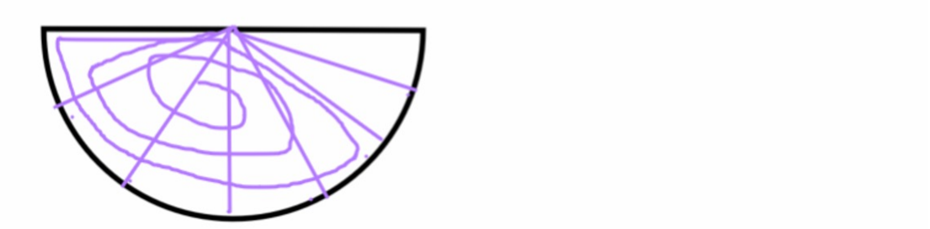
Schemes

3. The bar shown below is 7/3 as long as a whole candy bar. Draw the whole candy bar.



5. The stick shown below is 4/5 as long as a whole candy bar. Draw the whole candy bar.

2. The piece of pie below is 7/5 as big as your piece of pie. Draw your piece of pie.



6. The piece of pie below is 2/5 as big as your piece of pie. Draw your piece of pie.

Algebra

Which example could represent a linear function?

○

x	-3	0	3
y	4	6	8

○ $\frac{5}{x} + y = -7$

○

x	1	3	5	3
y	4	2	0	-2

○ $x + \frac{2}{y} = 4$

Conceptual Knowledge

Below is the beginning of Gabriella's, Jamal's, and Nadia's work in solving x + 7 - 3 = 12 - 2x.

Gabriella's way:	Jamal's way:	Nadia's way:
Subtract 3 from 7: $x + 4 = 12 - 2x$	Add $2x$ to both sides: $3x + 7 - 3 = 12$	Subtract $(7 - 3)$ from both sides: $x = 8 - 2x$

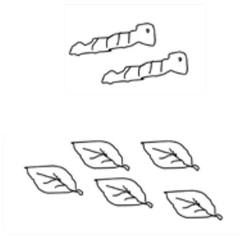
To start solving this problem, which way(s) may be used?

Flexibility

Solve the equation for y. Show your work on paper and type your answer here.

$5(y - 2) = -3(y - 2) + 4$

Procedural Knowledge



A class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would they need each day for 12 caterpillars?

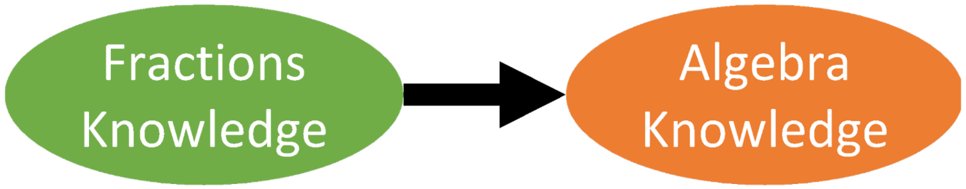
Proportional Reasoning

Overall score

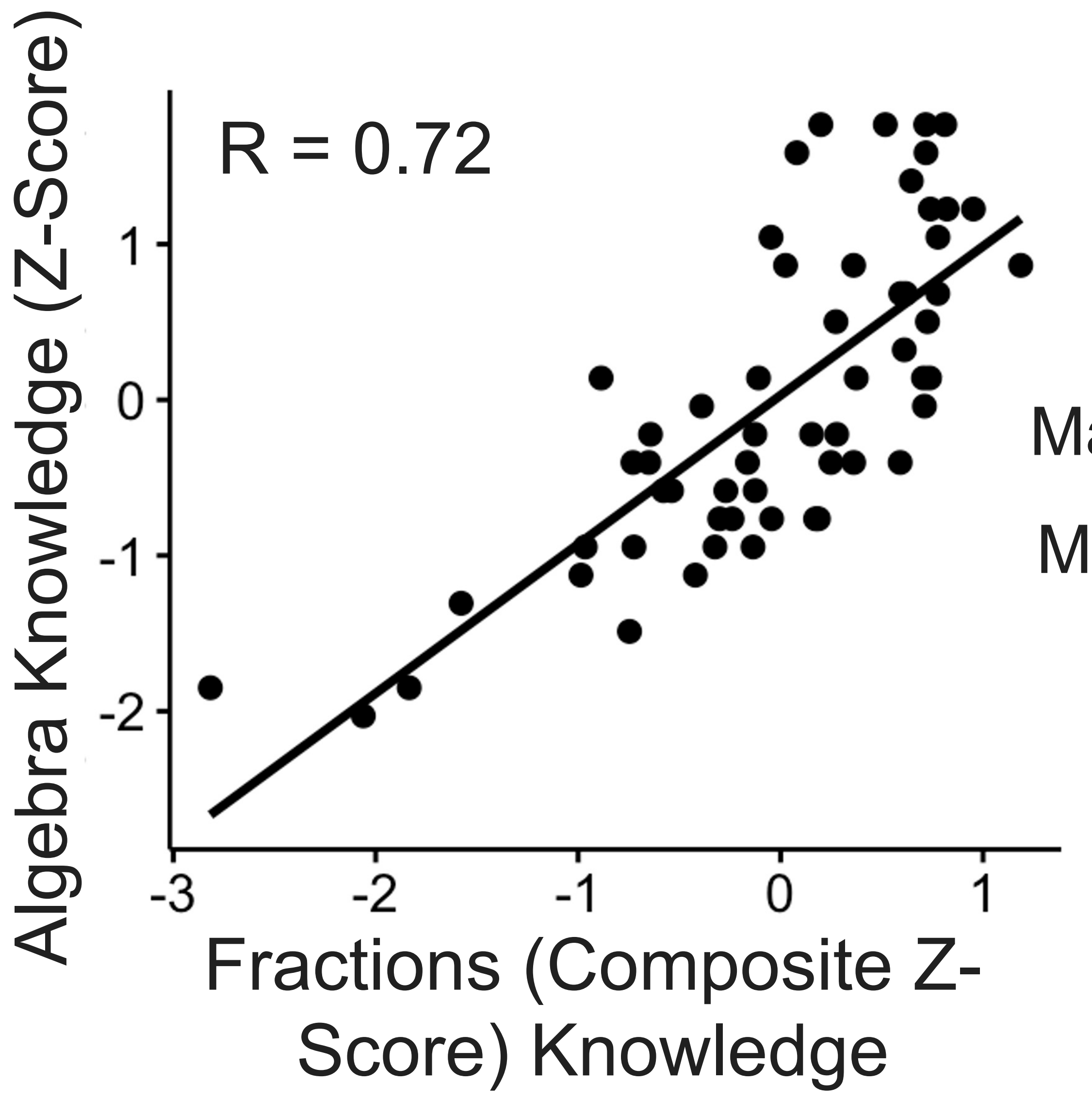
% Accuracy

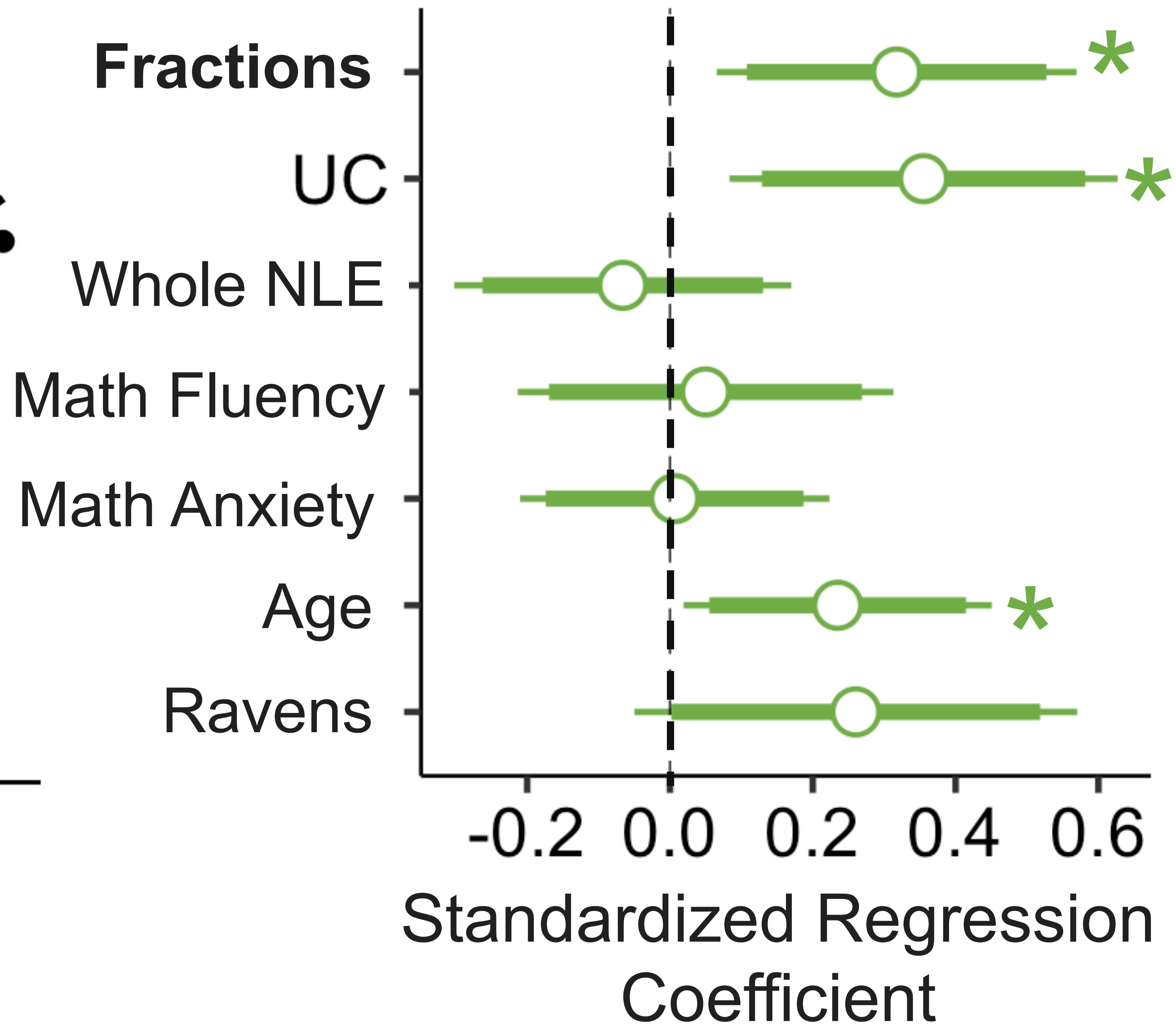
Hypotheses

H1. Overall fractions knowledge will predict algebra scores, even accounting for covariates.



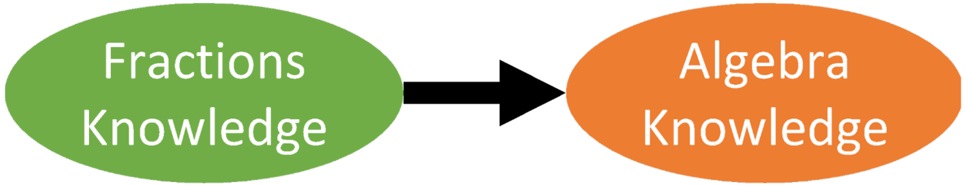
Overall fractions knowledge predicted algebra.



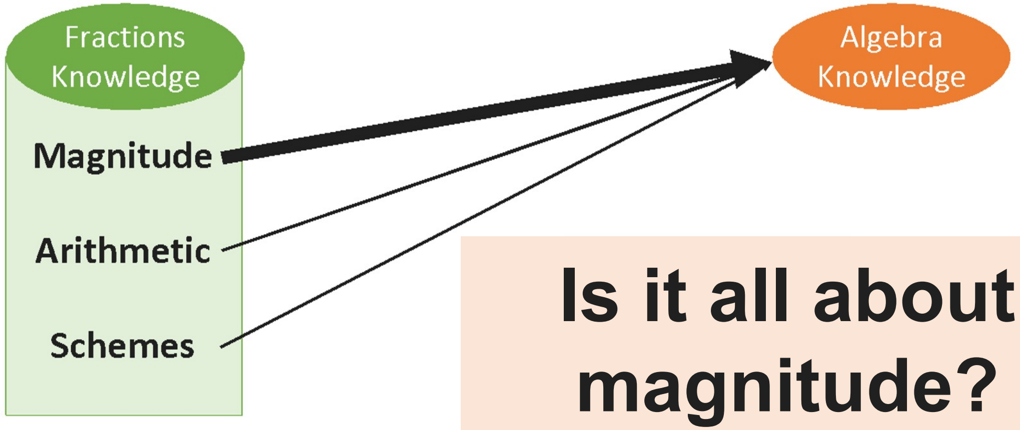


Hypotheses

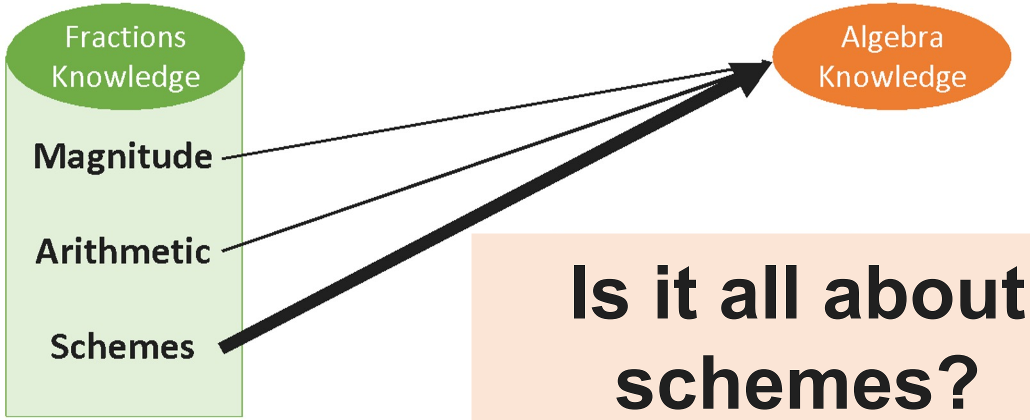
H1. Overall fractions knowledge will predict algebra scores, even accounting for covariates.



H2. Competing hypotheses about which aspect of fractions will be most important…

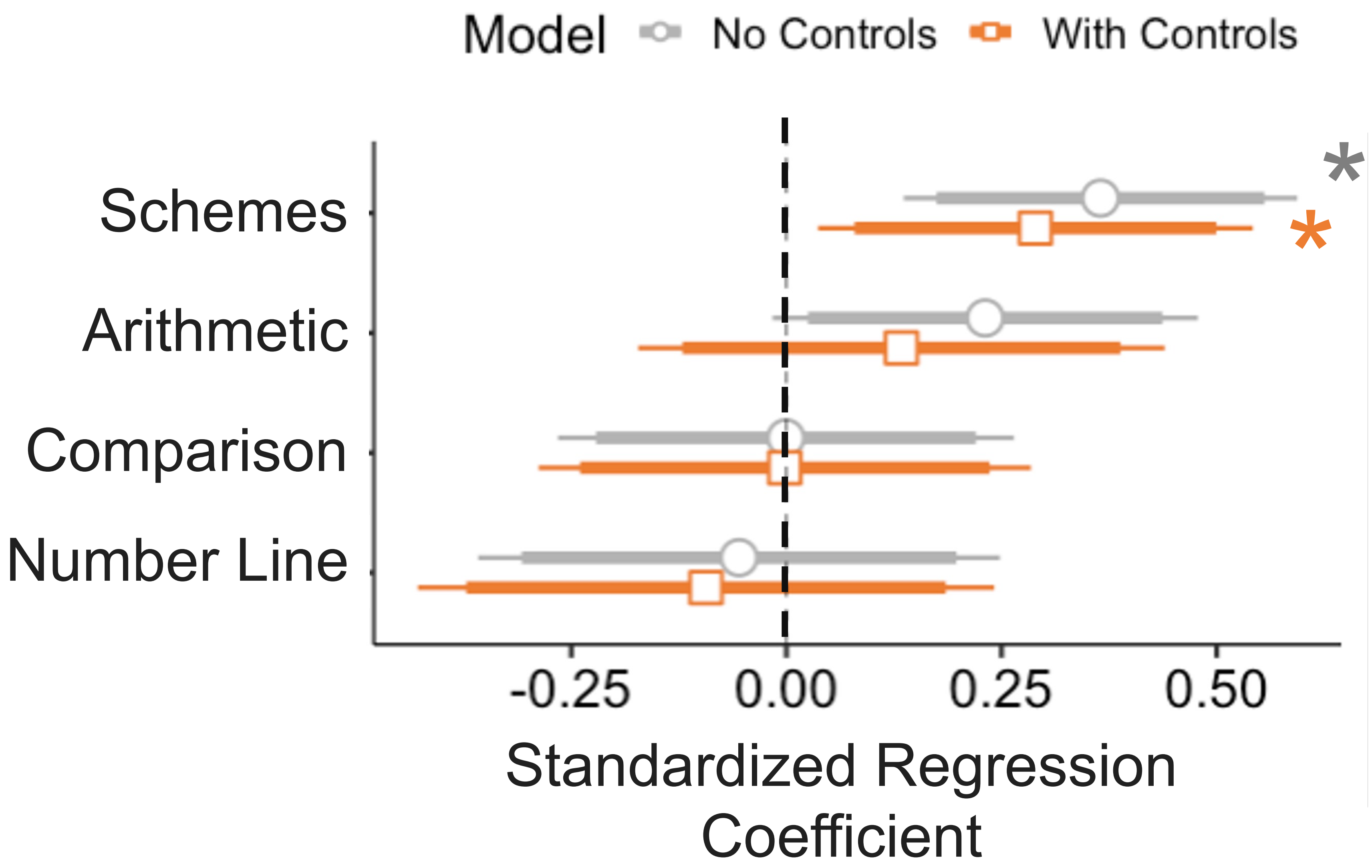


Is it all about magnitude?



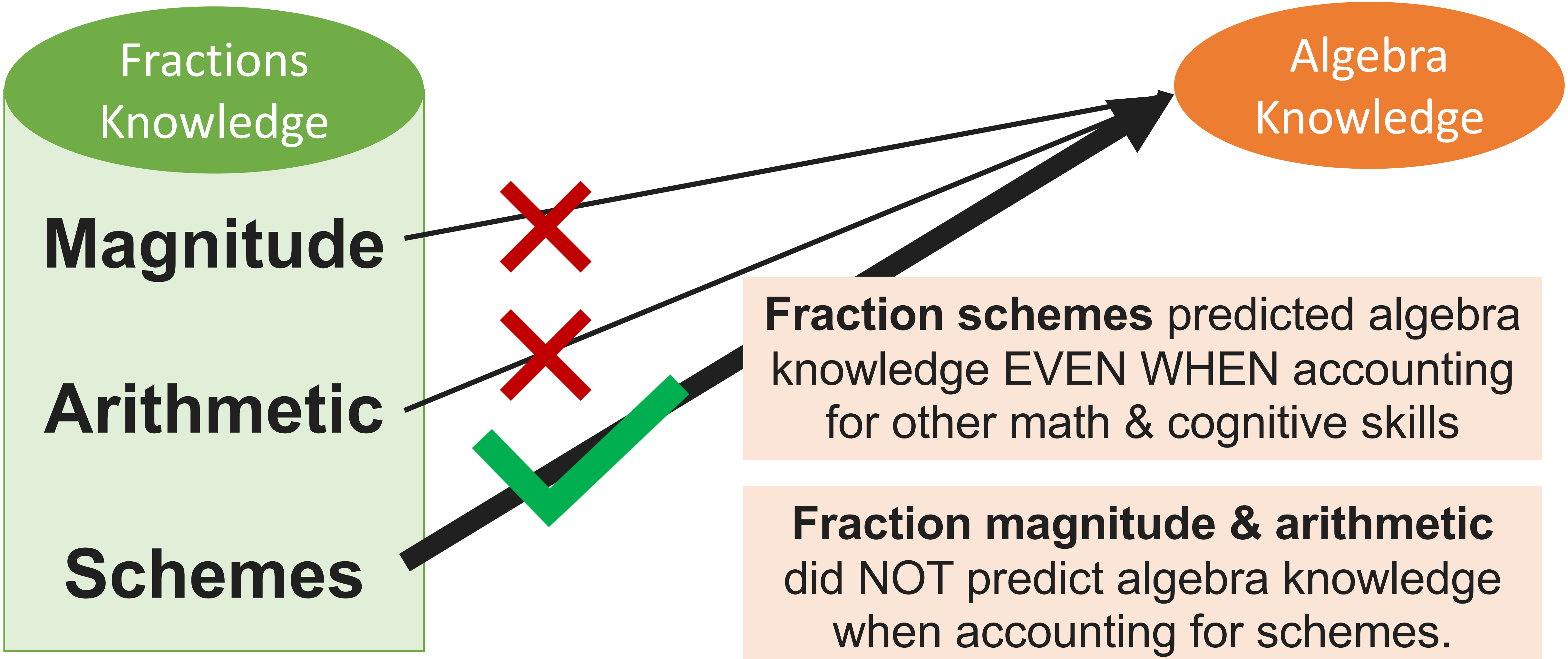
Is it all about schemes?

Which aspect(s) of fractions were UNIQUELY associated with algebra knowledge?



Only **fraction schemes** uniquely predicted students' algebra scores!

Summary of Study 2 Results



Fraction schemes predicted algebra knowledge EVEN WHEN accounting for other math & cognitive skills

Fraction magnitude & arithmetic did NOT predict algebra knowledge when accounting for schemes.

Future Directions

Further investigate mechanisms

3. The bar shown below is 7/3 as long as a whole candy bar. Draw the whole candy bar.

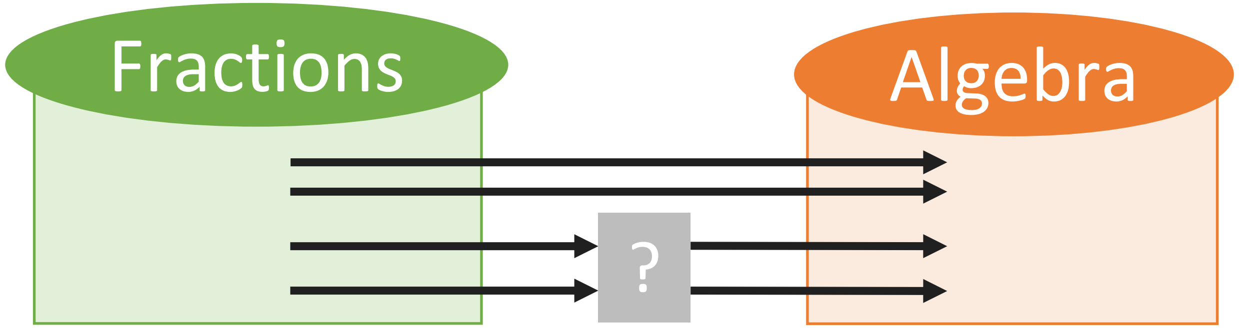
$$5(y\text{-}2) = \text{-}3(y\text{-}2) + 4$$

Replicate in different contexts

Moving Forward: Our Interdisciplinary Approach

1. Develop more comprehensive assessments of both fractions and algebra knowledge.
2. Track relations between multiple aspects of fractions and algebra longitudinally with 7th-9th grade students.

Move toward **specific & actionable** fractions-algebra models.



Thank you!

Percival Matthews

Ana Stephens

Allison Monday

Thank you to all participating students and families!

MELD Mathematics Education Learning & Development Lab

UNIVERSITY OF DELAWARE
EDUCATION & HUMAN DEVELOPMENT
Nancy Jordan's Fraction Lab

Undergraduate Research Assistants:

Valerie Buroker
Rose Eisenmenger
Amelia Jensen
Phoebe Miller
Yining Zhang

ies

JSMF

WARF
Wisconsin Alumni Research Foundation

Mechanisms: Fraction magnitude knowledge

$$2 = \frac{1}{4}x$$

Estimation and error checking

(Siegler et al., 2011)

$$\frac{1}{3} = \frac{2}{6} = \frac{10}{30}$$

Covariational and ***functional*** thinking

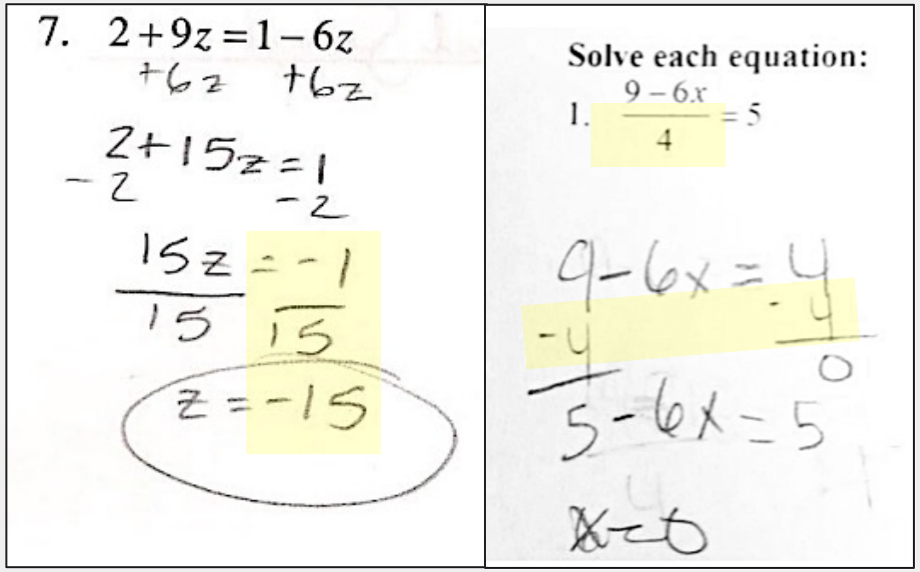
(DeWolf et al., 2015
Matthews & Ellis, 2018)

$$4b = 3$$

Flexible view of ***variables***

(Christou & Vosniadou, 2012)

Mechanisms: Fraction arithmetic



Fewer fraction-related errors in problem-solving

(Booth et al., 2014, *Journal of Problem Solving*)

$$2\frac{1}{2} \times 4 =$$

$$40 + 18 - 8 = x + 3$$

More **flexible** or efficient **strategies**

(Marghetis et al., 2016; Schneider et al., 2012; Silla et al., under review)

Mechanisms: Fraction schemes

3. The bar shown below is 7/3 as long as a whole candy bar. Draw the whole candy bar.

Children's **multiplicative reasoning** improves as they build increasingly complex fraction schemes.

(e.g., Boyce & Norton, 2016; Hackenberg, 2007; 2010; Hackenberg & Tillema, 2009; Steffe et al., 2010)

Theo has a stack of CDs some number of cm tall. Sam's stack is two-fifths of that height. Write an expression for how tall Sam's stack is.

$$S = \frac{2}{5} \times T \qquad T = \frac{5}{2} \times S$$

Multiplicative reasoning is crucial for understanding **relations between unknowns** in algebra.

(e.g., Eriksson & Sumpter, 2021; Hackenberg, 2013; Hackenberg & Lee, 2015)