Relations Between Grades 7-9 Students' Proportional Reasoning Strategies and Fraction Schemes

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Introduction

- Fraction knowledge is a challenge for students but is a predictor of algebra success and math achievment (Booth et al., 2014)
- More needs to be learned about the relationship between specific aspects of fractions and specifics aspects of algebra
- This study explores the theory that this relation can be understood through proportional reasoning
- The progression of fraction schemes is a framework for conceptualizing students' progression of fraction understanding (Wilkins & Norton, 2018)
- Proportional reasoning Involves multiplicative relationships considering how two linked quanitites vary and scale

Research Questions

What are the relations between grades 7-9 students' proportional reasoning strategies and fraction schemes?

a. Will students with higher fraction schemes scores be more likely to use a specific proportional reasoning strategy?

b. Do students that use more proportional reasoning strategies score higher on fraction schemes?

Methods

Participants

- 48 students in grades 7-9 (20 grade 7, 10 grade 8, and 18 grade 9)

Procedure

- All students participated in 4 Zoom sessions that measured their fractions and algebra knowledge using a variety of tasks
- In Session 2, students completed 16 tasks designed to measure their fraction schemes; they were asked to draw on the screen to show their thinking
- In Session 4, students were asked to explain their reasoning out loud when solving a variety of tasks, including 4 proportional reasoning tasks

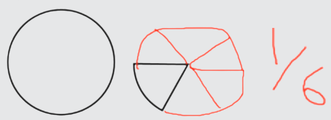
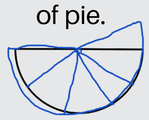
Measures

Proportional Reasoning

Example:

The Boston Park Committee is building parks. They found that 15 maple trees can shade 21 picnic tables when they built the Raymond Street Park. On Charles Street they will make a bigger park and can afford to buy 50 maple trees. How many picnic tables can be shaded at the new park?

Strategy	Example Solution
Build up or down	"15 trees can shade 21 tables, so 30 trees would shade 42 tables, 45 trees would shade 63 tables..."
Unit Factor	"15 trees per 21 tables, so I did 21 divided by 15. Then I mulitplied that by 50 for the amount of trees; that equals 210 divided by 3, which equals 70."
Formal Equation	"I set up a ratio of 15 over 21 to represent trees per tables. Then I did 50 over x and cross multiped to get 15x=1050 which I divided by 15 to get 70."

Fraction Schemes	Example Tasks
Part-whole scheme Produce proper fraction *m/n* by paritioning the whole into *n* parts and disembedding *m* of the parts	Make 1/9 of the candy bar shown below. 
Measurement scheme for unit fractions Determine size of unit fraction relative to an unpartitioned whole through iterating the unit fraction to produce the whole	What fraction is the smaller pie piece out of the whole pie? 
Measurement scheme for proper fractions Produce unknown whole from a proper fraction by partitioning into *m* parts and iterating unit fraction parts *n* times	The piece of pie below is 5/6 as big as your pice. Draw your piece of pie. 
Generalized measurement scheme for fractions Produce unknown whole from any fraction, including improper fractions, by partitioning into *m* parts and iterating unit fraction parts *n* times to find the whole	The bar shown below is 7/3 as long as a whole candy bar. Draw the whole candy bar. 

Scoring and Coding

- Each schemes item received a numeric score (1, 0.6, 0.4, 0) corresponding to strong indication (1) to strong counter-indication (0) that the participant operated in a manner consistent with the targeted scheme
- Each proportional reasoning item was scored for correctness and coded for strategy use

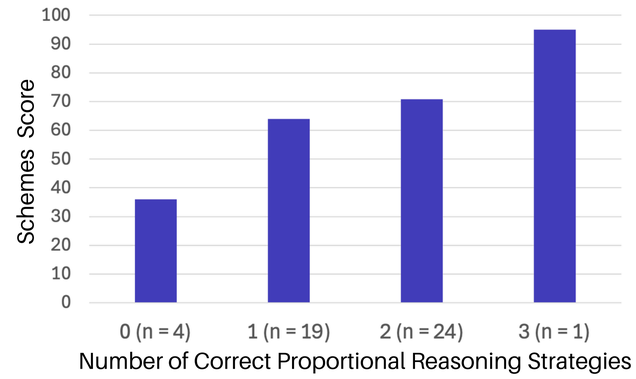
Results

a. Number of students using each proportional reasoning strategy performing above and below the median on fraction schemes

Strategy	Schemes score > median	Schemes score < median
Build up or down	21	18
Unit Factor	14	9
Formal Equation	5	4

Median score = 66.25%

b. Relation Between Number of Correct Proportional Reasoning Strategies and Schemes Performance



Schemes performance predicts number of proportional reasoning strategies when controlling for grade (p= .002)

Discussion

- Did not find evidence for a relationship between specific proportional reasoning strategies and schemes scores
- Strategy flexibility is a key component to explain the relationship between fraction schemes and proportional reasoning
  - Having stronger understanding of fractions may allow students to solve proportional reasoning problems in a variety of ways
- Future research should continue looking at other aspects of fractions and algebra to understand the relationship further

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